

## **Coexistence of UWB and Legacy Narrowband Systems**

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Discussion Draft*

### **Introduction**

The purpose of this paper is to discuss constraints on the coexistence of pulsed UWB systems and legacy military radios, which are “narrowband” relative to the bandwidth of the UWB pulse. The purpose here is not to provide detailed analysis of the UWB signal characteristics, or of the effect of the UWB signal on a narrowband receiver (such analysis is provided in [1]), but rather to examine the constraints on throughput and range of a UWB system that pertain to shared operation, given some stated maximum tolerable interference limit to the legacy system and the relevant system parameters (frequency, antenna gains, noise figures, etc.). An interference limit might be stated as a maximum allowed interference-to-noise ratio when the UWB transmitter is some minimum distance from the legacy receiver; e.g.,

$I/N \leq -3$  dB when the UWB transmitter is 10 meters from the legacy receiver, where  $I$  is the average UWB interference power experienced by the legacy system receiver, at the intermediate frequency (IF) output, and  $N$  is the thermal noise power at the same point.

Interference from the legacy narrowband systems to the UWB receiver is also briefly discussed.

### **UWB Communication Model**

Assume a UWB Communications System with bit rate  $R_b$  and a requirement that

$E_b/N_0 \geq (E_b/N_0)_{\min} = x$ , where  $E_b$  is the received energy per data bit and  $N_0/2$  is the two-sided noise power spectral density in watts/Hz. The minimum average received power at the UWB receiver is therefore

$$\bar{P}_{RX,uwb} = E_b R_b \geq R_b \cdot x \cdot N_0 \quad (1)$$

If  $l_{uwb}$  is the net path loss between the UWB transmitter and receiver, including antenna gains and systems losses, then the required UWB transmit power is

$$\bar{P}_{TX,uwb} = \bar{P}_{RX,uwb} \cdot l_{uwb} = x N_0 \cdot R_b l_{uwb} \quad (2)$$

that is, required transmit power is proportional to the product of data rate and path loss.

If  $l_{nb}$  is the path loss from the UWB transmitter to the narrowband (NB) receiver, then the total average UWB power at the NB receiver is  $\bar{P}_{TX,uwb} / l_{nb}$ .

The nature of the UWB interference as seen by the narrowband receiver will depend on (1) whether the average UWB pulse rate exceeds the IF bandwidth of the NB receiver; and (2) if so, how the pulses are positioned in time (uniformly spaced, pseudo-randomly dithered, modulated). While the details of the UWB signal impact on the NB receiver depends on these factors, probably the most useful general measure of the UWB impact is the average UWB power within the NB receiver IF bandwidth, which is

$$I = a(f_0) \frac{B_{nb}}{B_{uwb}} \cdot \frac{\bar{P}_{TX,uwb}}{l_{nb}} = a(f_0) \frac{B_{nb}}{B_{uwb}} \cdot x R_b \frac{l_{uwb}}{l_{nb}} N_0 \quad (3)$$

where  $f_0$  is the channel center frequency of the NB receiver,  $B_{nb}$  is the IF bandwidth of the NB receiver,  $B_{uwb}$  is the equivalent rectangular (one-sided) UWB bandwidth, and  $a(f_0)$  is the UWB pulse energy spectral density, relative to its maximum value, at frequency  $f_0$ .

The thermal noise of the NB receiver is  $N_{nb} = B_{nb} N_0$ , so the ratio of the average UWB interference to the thermal noise at the NB receiver is

$$\frac{I}{N} = a(f_0) \cdot x \cdot \frac{R_b}{B_{uwb}} \cdot \frac{l_{uwb}}{l_{nb}} \quad (4)$$

Note that this is independent of any of the parameters of the NB system, except its center frequency. The implicit assumption in this expression is that  $N_0$  is the same for the UWB and NB systems (their receivers have the same noise figure). The expression is easily generalized to account for different noise figures, using  $P_{TX,uwb} = x f_{uwb} kT \cdot R_b l_{uwb}$ , where  $f_{uwb}$  is the UWB noise factor (the noise figure is  $F_{uwb} = 10 \log f_{uwb}$ ),  $k$  is Boltzman's constant, and  $T$  is the reference noise temperature. Then

$$I = a(f_0) \frac{B_{nb}}{B_{uwb}} \cdot x R_b \frac{l_{uwb}}{l_{nb}} f_{uwb} kT \quad (5)$$

If  $f_{nb}$  is the noise factor of the NB system, then  $N_{nb} = B_{nb} f_{nb} kT$  and

$$\frac{I}{N} = a(f_0) \cdot x \cdot \frac{R_b}{B_{uwb}} \cdot \frac{l_{uwb}}{l_{nb}} \cdot \frac{f_{uwb}}{f_{nb}} \quad (6)$$

In dB, this becomes

$$(I/N)_{\text{dB}} = A(f_0) + X + 10 \log(R_b/B_{\text{uwb}}) + (L_{\text{uwb}} - L_{\text{nb}}) + (F_{\text{uwb}} - F_{\text{nb}}) \quad (7)$$

where upper-case quantities represent the dB values of the corresponding lower-case parameters; e.g.,  $A(f_0) = 10 \log a(f_0)$ ,  $L_{\text{uwb}} = 10 \log l_{\text{uwb}}$ , etc.

This relationship can be used to establish bounds on UWB system parameters, given a constraint on interference to legacy NB systems, in terms of a maximum value of  $I/N$  and a specified minimum loss  $L_{\text{nb}}$  between the UWB transmitter and the legacy NB receiver:

$$\underbrace{L_{\text{uwb}} + 10 \log\left(\frac{R_b}{B_{\text{uwb}}}\right)}_{\text{UWB rate/range performance}} = \underbrace{\left(\frac{I}{N}\right)_{\text{dB}}}_{\text{Legacy system protection criterion}} + L_{\text{nb}} + F_{\text{nb}} - A(f_0) - \underbrace{(X + F_{\text{uwb}})}_{\text{UWB receiver sensitivity}} \quad (8)$$

The loss  $L_{\text{nb}}$  between the UWB transmitter and the NB receiver will correspond to some specified distance  $d_{\text{nb}}$  which in general will be sufficiently small to use free-space path loss:

$$L_{\text{fs}} = 20 \log d + 20 \log f_0 - 27.6 \text{ dB} \quad (9)$$

where  $d$  is in meters and  $f_0$  is in MHz. The total loss  $L_{\text{nb}}$  would need to account for antenna gains and system losses.

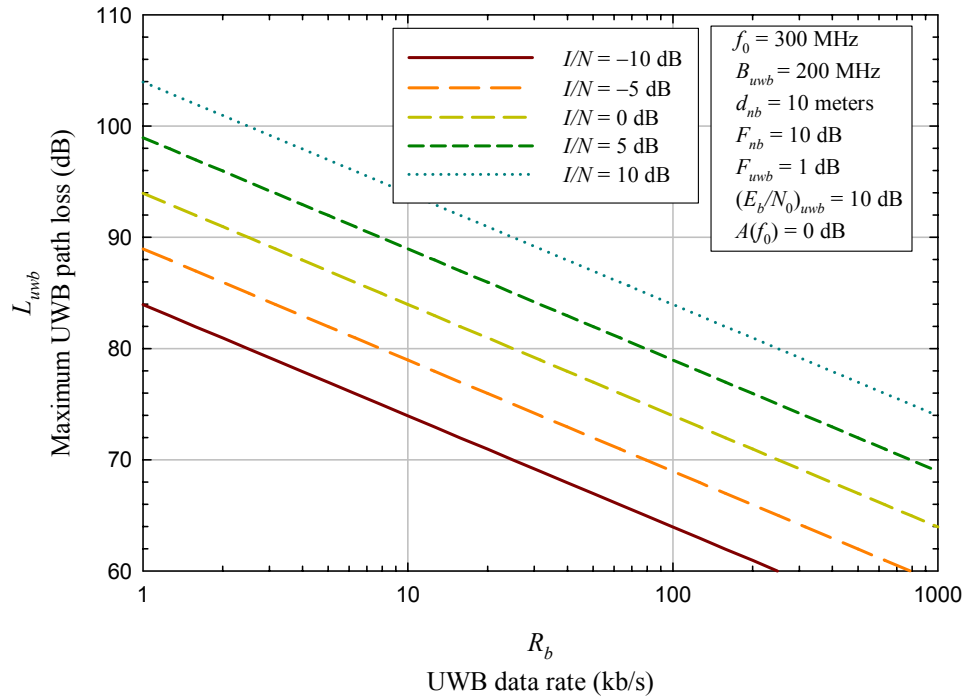
### **Examples**

Assume that  $d_{\text{nb}} = 10$  meters,  $X = 10 \log(E_b/N_0)_{\text{min}} = 10 \text{ dB}$ ,  $F_{\text{nb}} = 10 \text{ dB}$ , and  $F_{\text{uwb}} = 1 \text{ dB}$ . For a worst-case analysis,  $A(f_0) = 0 \text{ dB}$ . With these values,

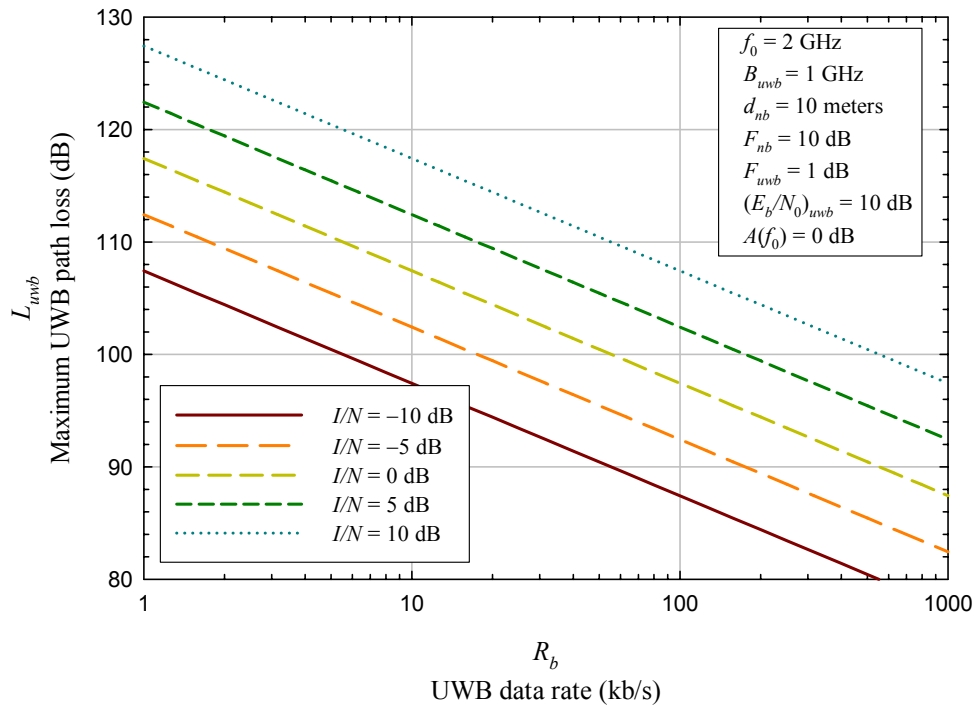
$$L_{\text{uwb}} = -10 \log R_b + (I/N)_{\text{dB}} + L_{\text{nb}} + 10 \log B_{\text{uwb}} - 1 \quad (10)$$

Further assume that the interference criterion is specified for a 10-meter separation between the UWB transmitter and NB receiver; that is,  $d_{\text{nb}} = 10$  meters. Finally, assume that the loss between the UWB transmitter and NB receiver is given by the free-space formula, and that system losses and antenna gains in the interference path sum to 0 dB.

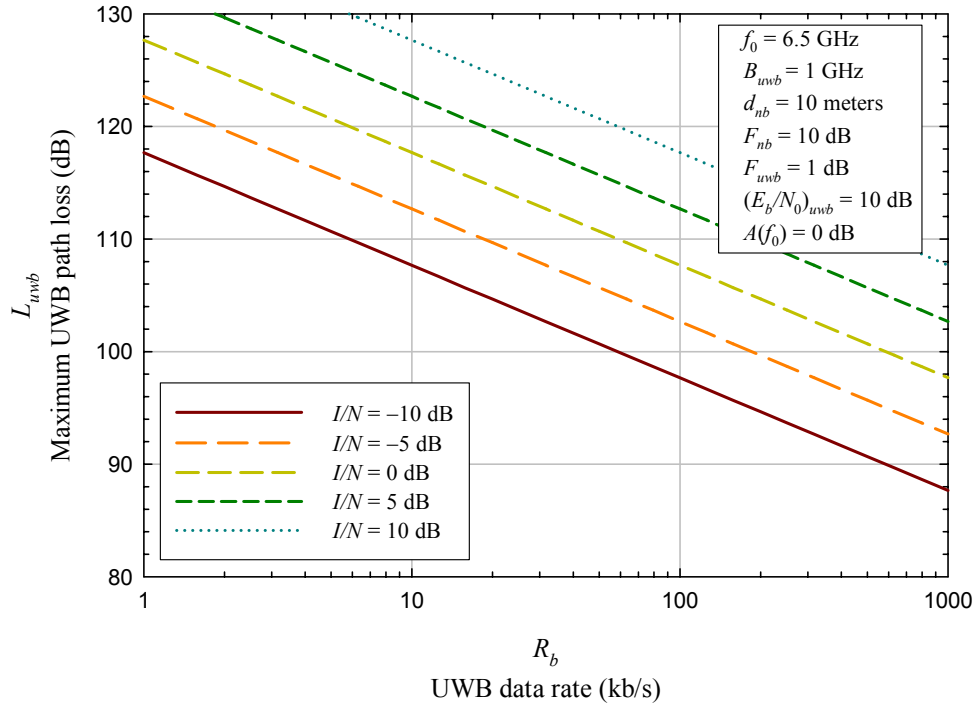
Given these assumptions, Figure 1 - Figure 3 show, for 3 different cases, the maximum loss  $L_{\text{uwb}}$  that can be sustained between the UWB transmitter and its companion receiver to maintain the specified  $E_b/N_0$  (10 dB) for the UWB link, for different values of  $I/N$  as seen by the legacy NB receiver. The only differences among the three figures are the UWB center frequency and bandwidth.



**Figure 1:** Maximum UWB path loss vs. UWB data rate to meet legacy system interference criteria with a 10-meter protection distance, with a 300-MHz center frequency and a 200-MHz UWB bandwidth.



**Figure 2:** Maximum UWB path loss vs. UWB data rate to meet legacy system interference criteria with a 10-meter protection distance, with a 2-GHz center frequency and a 1-GHz UWB bandwidth.



**Figure 3:** Maximum UWB path loss vs. UWB data rate to meet legacy system interference criteria with a 10-meter protection distance, with a 6.5-GHz center frequency and a 1-GHz UWB bandwidth.

The next step is to translate the UWB path loss that can be supported into an operating range. There are many different propagation models, and which are appropriate will depend on the parameters, including frequency, distance, antenna elevations, and type of terrain. A simple model will be used here for illustrative purposes.

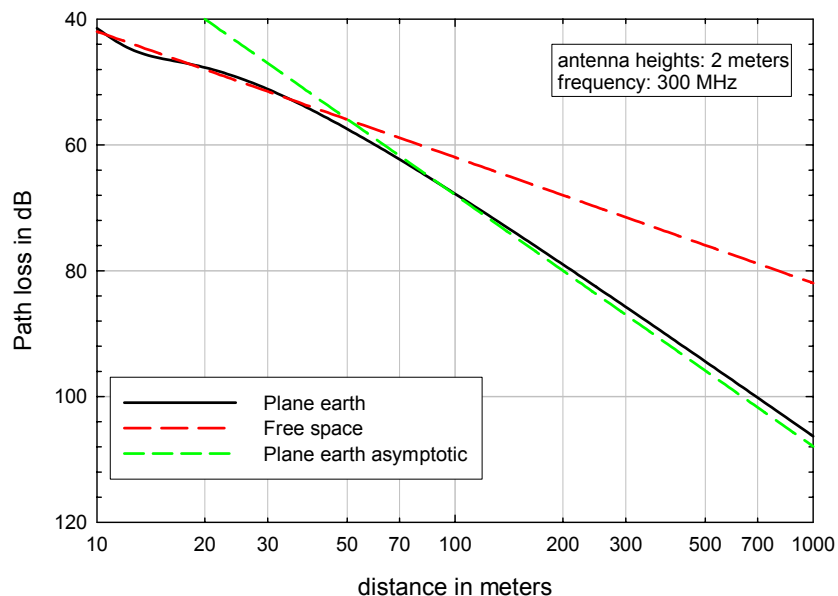
Figure 4 shows curves for the free space, plane earth and asymptotic plane earth models, for a frequency of 300 MHz. Figure 5 and Figure 6 show similar curves for 2 GHz and 6.5 GHz, respectively.

The free space model was discussed above. The plane earth model assumes direct and ground-reflected rays which add coherently at the receiver, and the phase relationship between the two depends on the difference in the two path lengths (relative to a wavelength). This accounts for the oscillatory behavior seen for the plane earth model. The asymptotic plane earth path loss, for  $d^2 \gg h_1 h_2$ , is

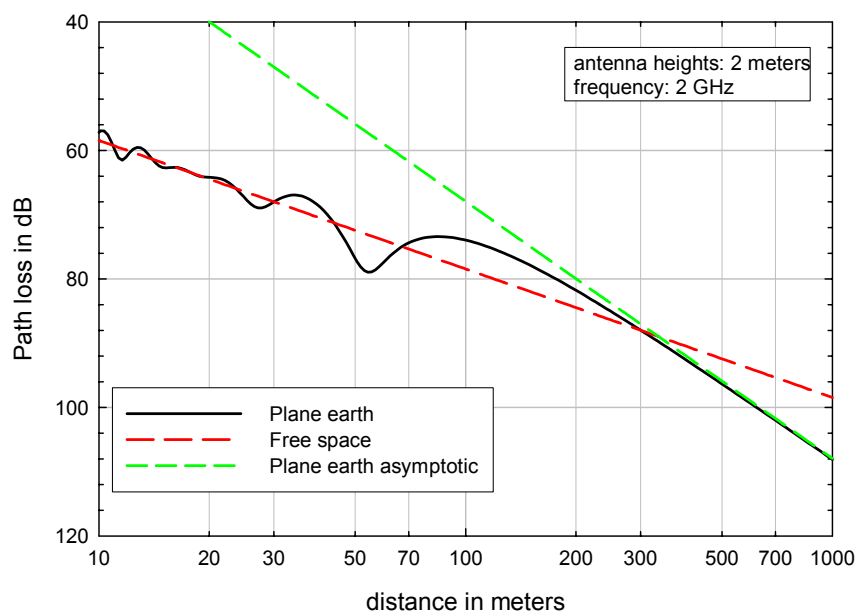
$$L_{pe,asymp} = \frac{d^4}{(h_1 h_2)^2} \quad (11)$$

A reasonable approximation to the plane-earth model is a combination of the free space and approximate plane earth models, which forms a “dual slope” model with a breakpoint  $d_b$  such that  $(4\pi d_b/\lambda)^2 = (d_b^2/h_1 h_2)^2$ , or  $d_b = 4\pi h_1 h_2/\lambda$ , giving the model:

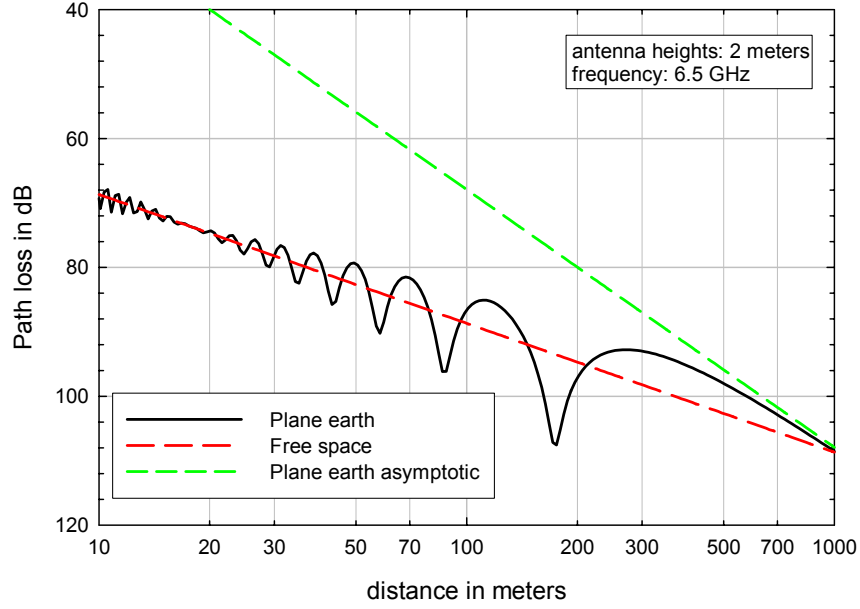
$$L_{ds} = \begin{cases} (4\pi d/\lambda)^2 & d \leq 4\pi h_1 h_2/\lambda \\ (d^2/h_1 h_2)^2 & d > 4\pi h_1 h_2/\lambda \end{cases} \quad (12)$$



**Figure 4:** Free space and plane earth path loss at 300 MHz.



**Figure 5:** Free space and plane earth path loss at 2 GHz.



**Figure 6:** Free space and plane earth path loss at 6.5 GHz.

Note that the higher the frequency, the greater the breakpoint distance.

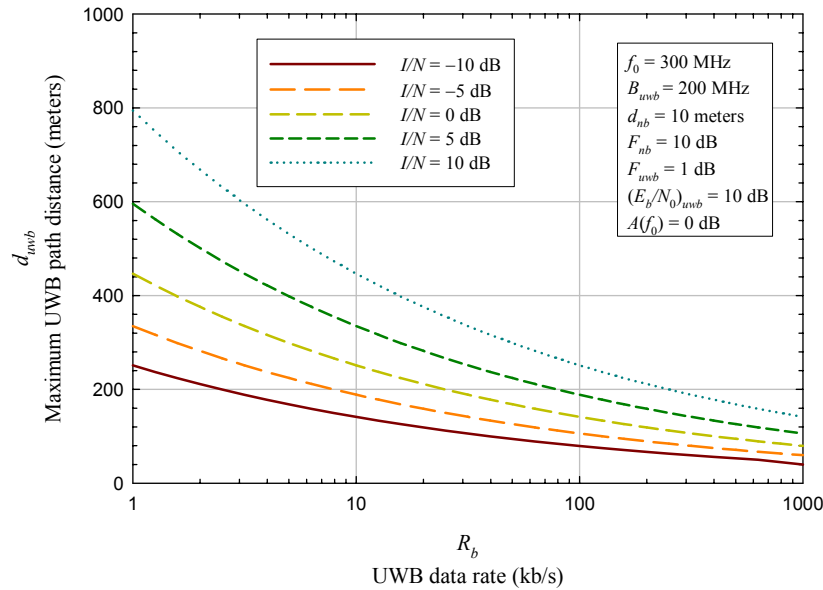
For the current problem, what is needed is to determine  $d$  given the loss  $L$ :

$$20\log d = \begin{cases} L - 20\log f + 27.6 & L \leq L_b \\ L/2 + 10\log(h_1 h_2) & L > L_b \end{cases} \quad (13)$$

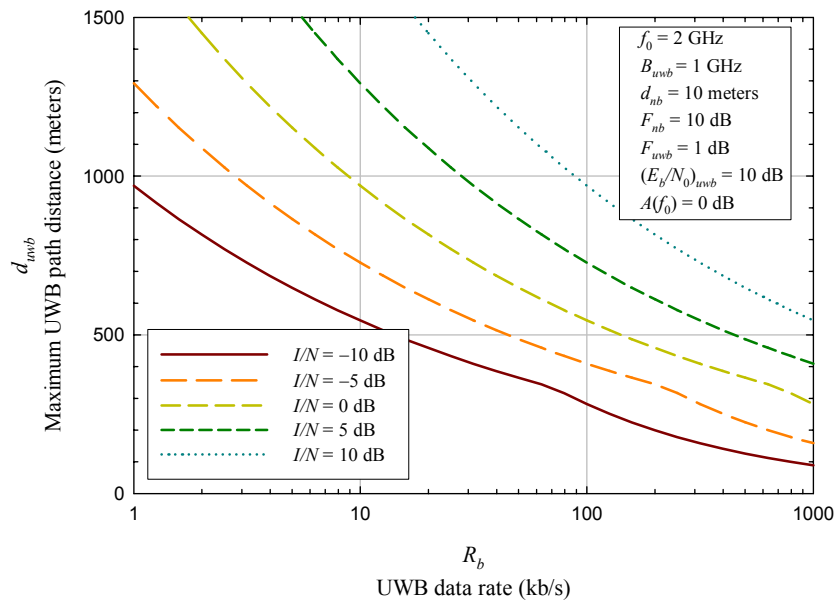
where the loss break-point is

$$L_b = 20\log h_1 h_2 + 40\log f - 55.2 \quad (14)$$

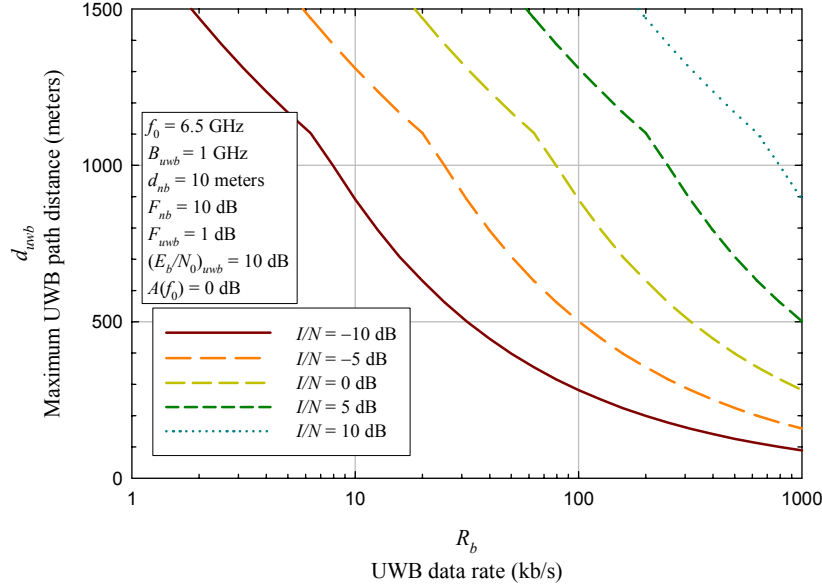
Applying this path loss model to the case shown in Figure 1 yields Figure 7 ( $f_0 = 300$  MHz,  $B_{uwb} = 200$  MHz). Similarly, Figure 8 corresponds to Figure 2 ( $f_0 = 2$  GHz,  $B_{uwb} = 1$  GHz), and Figure 9 to Figure 3 ( $f_0 = 6.5$  GHz,  $B_{uwb} = 1$  GHz).



**Figure 7:** Maximum UWB range vs. UWB data rate to meet legacy system interference criterion with a 10-meter protection distance, for a 300 MHz center frequency and 200 MHz bandwidth.



**Figure 8:** Maximum UWB range vs. UWB data rate to meet legacy system interference criterion with a 10-meter protection distance for a 2 GHz center frequency and 1 GHz bandwidth.



**Figure 9:** Maximum UWB range vs. UWB data rate to meet legacy system interference criterion with a 10-meter protection distance for a 6.5 GHz center frequency and 1 GHz bandwidth.

### Antenna Gains and System Losses

The terms  $L_{uwb}$  and  $L_{nb}$  were defined as including antenna gains and system losses. To see the effect of these factors on the range-rate tradeoff, it is worthwhile to show them explicitly, at the cost of slightly more complexity in the expression. The net UWB path loss between the UWB transmitter and its associated receiver can be expressed as:

$$L_{uwb} = L_{p,uwb} - G_{TX,uwb} - G_{RX,uwb} \quad (15)$$

where  $L_{p,uwb}$  is the propagation path loss, and  $G_{TX,uwb}$  and  $G_{RX,uwb}$  are the transmit and receive UWB antenna gains, including any associated system losses (there is no need to separate RF-path system losses from the antenna gains in the link budget equations).

Similarly, in the UWB-to-NB (interference) path,

$$L_{nb} = L_{p,nb} - G_{TX,uwb \rightarrow nb} - G_{RX,nb} \quad (16)$$

where  $L_{p,nb}$  is the propagation path loss between the UWB transmitter and the NB receiver, as seen by the NB receiver with center frequency  $f_0$  (free-space loss at 10 meters, in the above examples). The term  $G_{TX,uwb \rightarrow nb}$  represents the UWB transmit antenna gain as seen by the NB receiver. This may be quite different than the gain  $G_{TX,uwb}$  seen by the UWB receiver for two reasons. First, the NB receiver will in general be located in a different direction from the

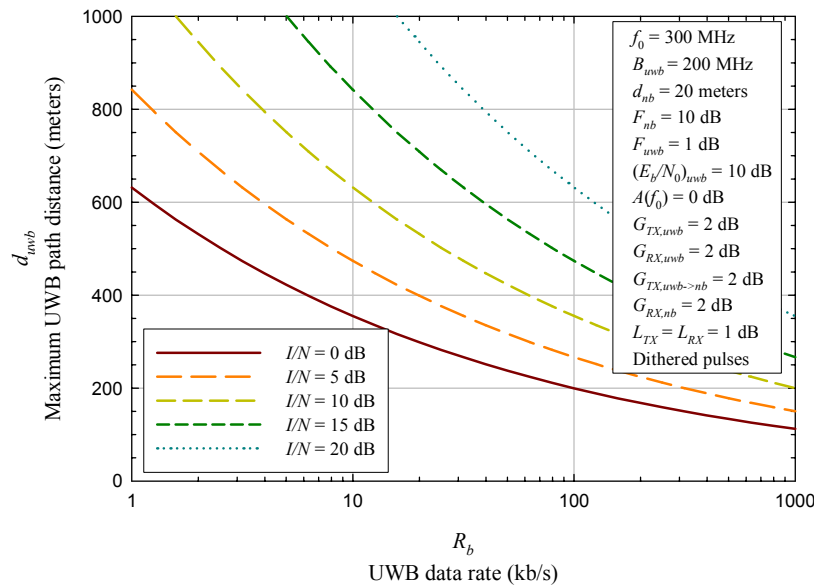
UWB transmitter than the UWB receiver. Second, the UWB transmit antenna pattern may appear different to wideband and narrowband signals. Finally,  $G_{RX,nb}$  is the net gain (again, including losses) of the narrowband receiver antenna in the direction of the UWB transmitter.

Substituting (15) and (16) into (8) gives:

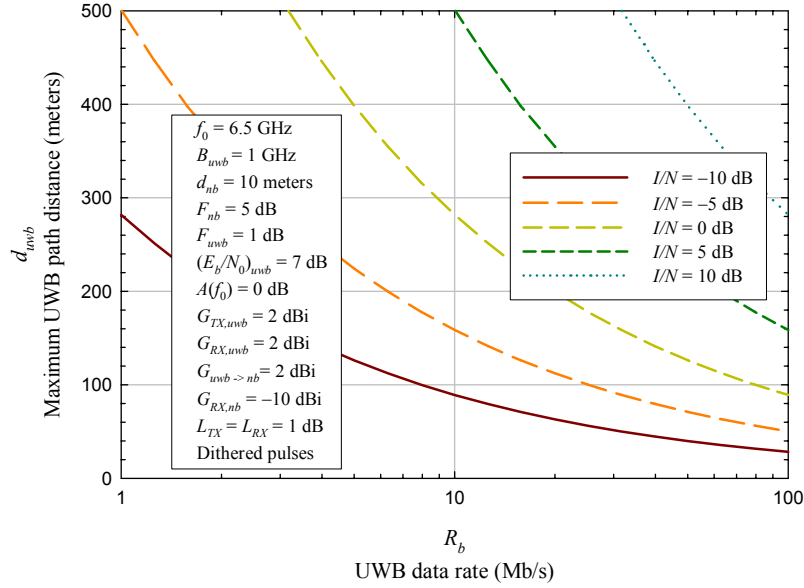
$$\underbrace{L_{p,uwb} + 10 \log \left( \frac{R_b}{B_{uwb}} \right)}_{\text{UWB rate/range performance}} = \underbrace{\left( \frac{I}{N} \right)_{\text{dB}}}_{\text{Legacy system protection criterion}} + L_{p,nb} + F_{nb} - A(f_0) - \underbrace{(X + F_{uwb})}_{\text{UWB receiver sensitivity}} + \underbrace{G_{TX,uwb} + G_{RX,uwb} - G_{TX,uwb \rightarrow nb} - G_{RX,nb}}_{\text{net effect of antenna gains and system losses}} \quad (17)$$

As can be seen, the net antenna gain effect (the sum of the final four terms) simply shifts, dB-for-dB, the value of  $L_{p,uwb}$  corresponding to a given  $R_b/B_{uwb}$ .

Figure 10 shows range/rate curves, accounting for antenna gain, for a VHF/UHF UWB communication system. With the system delivering 10 kb/s and a 500-meter range,  $I/N \cong 6$  dB to a narrowband legacy receiver 20 meters from the UWB transmitter. Figure 11 shows an example for a high-rate UWB communication system operating at 6.5 GHz. In this case, the UWB system can achieve a rate of 10 Mb/s at a range of about 100 meters, with  $I/N = -10$  dB for the legacy receiver that is 10 meters from the UWB transmitter.



**Figure 10:** Handheld VHF/UHF UWB comm system, 10 kb/s at 500 m.



**Figure 11:** High-rate UWB comm system, 10 Mb/s at 100m.

### UWB Radar Model

If the UWB system is being used as a radar with pulse rate  $R_p$  and received energy per pulse  $E_p$ , and a requirement that  $E_p/N_0 \geq (E_p/N_0)_{\min} = y$ . Therefore, the lower bound on the average received power is

$$P_{RX,uwb} \geq R_p \cdot y \cdot N_0 \quad (18)$$

and the required transmit power is

$$P_{TX,uwb} = P_{RX,uwb} \cdot l_{2uwb} \quad (19)$$

where  $l_{2uwb}$  is the two-way path loss to the target and back, including antenna gains and system losses as well as the reflection from the target cross-section. The derivation of the interference-to-noise ratio at the NB receiver is similar to the earlier case, except  $R_b$  is replaced by  $R_p$  and  $L_{uwb}$  is replaced by the two-way dB path loss  $L_{2uwb}$ , giving a result similar to (8):

$$\underbrace{L_{2uwb} + 10 \log \left( \frac{R_p}{B_{uwb}} \right)}_{\text{UWB rate/range performance}} = \underbrace{\left( \frac{I}{N} \right)_{\text{dB}} + L_{nb}}_{\text{Legacy system protection criterion}} + F_{nb} - A(f_0) - \underbrace{(Y + F_{uwb})}_{\text{UWB receiver sensitivity}} \quad (20)$$

With the traditional radar equation, free-space propagation is assumed in both directions, and

$$I_{2uwb} = \frac{(4\pi)^3 d_{uwb}^4}{\sigma^2 \lambda^2} \quad (21)$$

where  $d_{uwb}$  is the distance from the radar to the target,  $\sigma$  is the radar cross-section of the target ( $m^2$ ),  $G = 10 \log g$  is the UWB radar antenna gain including losses (assumed the same in both directions), and  $\lambda$  is the wavelength of the radar signal. The propagation path loss (not including antenna gain or system losses) is

$$L_{p,2uwb} = 40 \log d_{uwb} + 20 \log f - 10 \log \sigma - 16.6 \text{ dB} \quad (22)$$

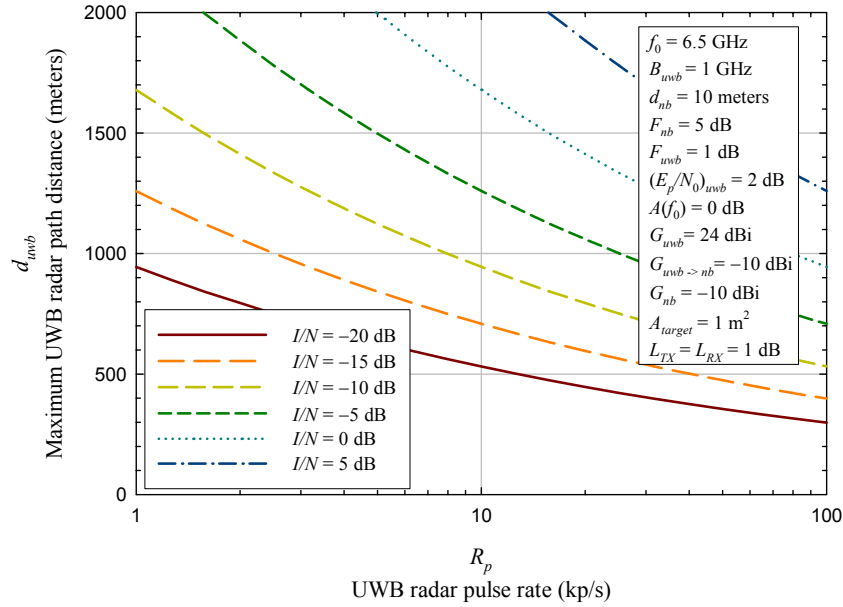
If antenna gains are shown explicitly, then (20) becomes:

$$\underbrace{L_{p,2uwb} + 10 \log \left( \frac{R_p}{B_{uwb}} \right)}_{\text{UWB rate/range performance}} = \underbrace{\left( \frac{I}{N} \right)_{\text{dB}} + L_{p,nb} + F_{nb} - A(f_0)}_{\text{Legacy system protection criterion}} - \underbrace{(Y + F_{uwb})}_{\text{UWB receiver sensitivity}} \quad (23)$$

$$+ \underbrace{G_{TX,uwb} + G_{RX,uwb} - G_{TX,uwb \rightarrow nb} - G_{RX,nb}}_{\text{net effect of antenna gains and system losses}}$$

Again, the antenna gains also include the effects of associated system losses.

Figure 12 shows the UWB radar range  $d_{uwb}$  vs. the pulse rate  $R_p$  for parameters listed. Note that the threshold value of  $E_p/N_0$  was set to 2 dB, corresponding to a case in which multiple pulses are integrated at the receiver. It was assumed that the UWB transmit and receive antenna gain was 24 dBi minus 1 dB for system losses. The NB receive gain in the direction of the UWB transmissions, and over the NB bandwidth, was taken as -10 dBi.



**Figure 12:** Maximum UWB range vs. UWB data rate to meet legacy system interference criterion with a 10-meter protection distance for a 6.5 GHz center frequency and 1 GHz bandwidth.

### Narrowband to UWB Interference

From the above results, it appears that UWB transmitters generally can be allowed sufficient transmit power to achieve a reasonable range/rate design tradeoff without causing excessive interference to legacy narrowband receivers. This is primarily due to the fact that only a small fraction of the UWB pulse energy falls within the legacy receiver signal bandwidth.

The reverse situation – interference from NB transmitters to UWB receivers – is more problematic. First, the total power of a NB transmission generally will fall within the UWB passband. Second, a wide UWB passband (several hundred MHz or more) may span multiple NB transmitters, some of which may be very powerful and/or very near to the UWB receiver.

To quantify this situation, consider a UWB receiver at which the average desired signal power is  $\bar{S}$ , the thermal noise is  $N$ , and the interference from the narrowband transmitter is  $I$  (intentional jamming is ignored here). As above, the UWB bandwidth and bit rate are  $B_{uwb}$  and  $R_b$ , and  $N_0$  represents the average PSD of the noise plus interference across the UWB passband; that is,

$$N_0 = \frac{N + I}{B_{uwb}}. \quad (24)$$

With  $E_b$  representing the received energy per data bit, then

$$\frac{E_b}{N_0} = \frac{B_{uwb}}{R_b} \cdot \frac{\bar{S}}{N + I} \quad (25)$$

If  $\tau_p$  is the effective pulse width, then the energy per pulse is (by definition; see the Annex),

$$E_p = S_{peak} \tau_p \quad (26)$$

where  $S_{peak}$  is the peak pulse power. Also,

$$\tau_p = \frac{k_{B\tau}}{B_{uwb}} \quad (27)$$

where  $k_{B\tau}$  is a constant on the order of 1 that depends on the pulse waveform shape. For simplicity, it is assumed here that  $k_{B\tau} = 1$ .

If the pulse rate is  $R_p$ , then there are  $K_{pb} = R_p / R_b$  pulses per bit and  $E_b = K_{pb} E_p$ . Since

$$\bar{S} = E_p R_p = S_{peak} \tau_p R_p = S_{peak} \frac{R_p}{B_{uwb}} = S_{peak} K_{pb} \frac{R_b}{B_{uwb}}, \quad (28)$$

it follows that

$$\frac{E_b}{N_0} = K_{pb} \frac{S_{peak}}{N + I} \quad (29)$$

From a detection perspective,  $K_{pb} = 1$  is probably the most efficient, unless multiple pulses can be coherently combined. Of course, if error-correcting codes are used with one pulse per coded bit, then  $K_{pb}$  will be the inverse of the code rate.

The required value of  $E_b / N_0$  will depend on a number of factors, including the necessary bit error rate (BER), the modulation/demodulation, and coding. Generally, the required  $E_b / N_0$  can be expected to be in the range of roughly 5 to 15 dB.

As an example of NB interference impact, consider the VHF/UHF UWB communication system (300 MHz center frequency, 200 MHz bandwidth, 10 kb/s data rate). If its noise figure is 1 dB and it requires that  $E_b / N_0 = 10$  dB, then with thermal noise only, the minimum average received power is

$$\bar{S} \geq 10 \log E_b + 10 \log R_b = -174 + 1 + 10 + 40 = -123 \text{ dBm} \quad (30)$$

and the corresponding peak power at the receiver is

$$S_{peak} = \bar{S} + 10 \log(B_{uwb} / R_b) = -80 \text{ dBm} \quad (31)$$

Now consider a 10-watt legacy narrowband transmitter 20 meters from the UWB receiver. The free-space path loss at 300 MHz and 20 meters is 48 dB. With 2-dB antenna gains and 1-dB system losses at each end, the interference power into the UWB receiver is -6 dBm.

Clearly, power received from the UWB transmitter would need to be increased by 84 dB, compared to its noise-limited value, to overcome the interference from the nearby narrowband transmitter. This means that the received average and peak power levels would need to be -39 dBm and +4 dBm, respectively.

If the UWB communication system is to have a range of 500 meters, the path loss at 300 MHz, with 2-meter transmit and receive antenna elevations, is about 96 dB (using the asymptotic plane earth model). With 2-dB antenna gains and 1-dB system losses at each end, a peak transmit power of 14 dBm would be needed to deliver -80 dBm received power, which is adequate to overcome thermal noise. The 84-dB increase required to overcome the narrowband interference corresponds to a peak transmit power of 98 dBm, or 6.3 MW. For a 100-meter UWB operating range, the required peak transmit power would be 70 dBm, or 10 kW. The average transmit power levels would be 55 dBm (316 W) for a 500-meter distance, and 27 dBm (0.5 W) for a 100-meter distance. Clearly, overcoming the narrowband interference by increasing transmit power is impractical, and in any event, it would cause increased interference to the legacy system receivers.

In sum, the received narrowband interference is 84 dB above the UWB receiver noise floor (-90 dBm), requiring a corresponding increase in received signal power. While this is but a single example with an arbitrary (but reasonable) set of parameters, the point is that narrowband interference can easily be many tens of dB above the UWB receiver noise floor. This suggests that UWB receivers should include the capability to either excise or avoid the narrowband interference. Excision might be accomplished using a frequency-selective receiver. Avoidance could possibly be realized in some cases by some form of frequency agility. These clearly are topics for further study.

### **Remarks about Assumptions**

Some simplifying assumptions have been made here in the interest of highlighting the general relationships and tradeoffs that relate to UWB/NB coexistence.

- Interference has been quantified as the average interference from the UWB signal that appears within the NB legacy system passband. While this is important, the exact impact of the interference will depend on its temporal and statistical characteristics. It may

appear impulsive, if the receiver IF bandwidth exceeds the pulse rate. If the IF bandwidth is less than the pulse rate and the pulses are regularly spaced in time, the interference will appear as a CW tone with power proportional to the square of the pulse rate. The location of the tone within the passband in general determines the effect on the victim NB legacy receiver performance. Finally, if the IF bandwidth is less than the pulse rate and the pulses are dithered, the interference at the NB IF will appear noise-like, and its statistical behavior will determine the exact effect on the receiver performance.

- Aggregate interference effects have been ignored.
- It has been assumed that the victim NB receiver center frequency is located at the maximum of the UWB pulse energy spectral density, resulting in worst-case interference. In fact, it is possible, using either pulse waveshaping techniques such as doublets, or specifically-tailored pulse repetition and dithering algorithms, to deliberately reduce the interference power from the UWB signal in a specific band. It is even conceivable the “smart” UWB systems could adapt to nearby NB systems to minimize interference. For example, doublet spacing could be adjusted to place a null at a desired frequency.
- The UWB receiver has been assumed ideal, and the only impairment in the calculations was thermal noise. In practice, the UWB receiver will be subject to interference from a multitude of NB transmissions, which must be taken into account for a complete analysis. Adaptive frequency-based UWB receiver structures is an area for further work. This approach would effectively allow the UWB receiver to “blank” frequency bands with high NB interference. This technique could be used together with a “smart” UWB transmitter as described above, which would put low signal energy into the affected band. Interference in both directions thereby could be minimized.
- The gain pattern of a UWB antenna, as seen by a NB receiver, needs to be understood. An antenna that appears highly directive to the UWB signal may have quite a different pattern from the perspective of the narrowband receiver.
- Narrowband propagation models have been used here to calculate the path loss for the UWB system. The inaccuracies associated with this simplification need to be understood.

## Annex Peak Pulse Power and Effective Pulse Duration

UWB system power output and interference to a narrowband system can also be expressed in terms of peak pulse power. The purpose of this Annex is to develop the relationships between peak pulse power and the expressions used in the body of this paper, which are based on energy per pulse or energy per data bit.

### Pulse Energy, Bandwidth, and Normalization

Let  $x(t)$  be a bandpass waveform (i.e.,  $\int_{-\infty}^{\infty} x(t)dt = 0$ ) normalized so that its energy and bandwidth are unity:

$$E_x = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} |X(f)|^2 df = 1 \qquad B_x = \frac{E_x}{2|X(f)|_{\max}^2} = 1$$

where  $X(f)$  is the Fourier transform of  $x(t)$ .

Such a normalized waveform can be formed from any finite-energy waveform  $y(t)$  which has energy  $E_y$  and bandwidth  $B_y$ . Define  $z(t) = y(t/B_y)$ , so that  $Z(f) = B_y \cdot Y(B_y f)$ , and  $E_z = B_y E_y$ . Thus, the bandwidth is normalized since

$$B_z = \frac{E_z}{2|Z(f)|_{\max}^2} = \frac{B_y E_y}{2B_y^2 |Y(f)|_{\max}^2} = \frac{1}{B_y} \cdot \frac{E_y}{2|Y(f)|_{\max}^2} = 1$$

To normalize the energy, let  $x(t) = z(t)/\sqrt{E_z}$  so that  $E_x = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} z^2(t)dt / E_z = 1$ .

Therefore, the complete normalization is:

$$x(t) = \frac{1}{\sqrt{B_y E_y}} y(t/B_y).$$

The normalized waveform  $x(t)$  specifies the pulse “shape”. If the actual pulse  $p(t)$  is to have energy  $E_p$  and bandwidth  $B_p$ , then

$$p(t) = \sqrt{B_p E_p} x(B_p t)$$

If the peak pulse power is  $P_{pk} = p_{\max}^2$ , then an effective pulse width could be defined as:

$$\tau_p \equiv \frac{E_p}{P_{pk}}$$

i.e., the duration of a rectangular pulse with amplitude  $p_{\max}$  and energy  $E_p$ . Note that the effective pulse width of  $x(t)$  is  $\tau_x = 1/x_{\max}^2$ , and therefore,  $\tau_p = \tau_x/B_p$ ; the effective pulse width is inversely proportional to the bandwidth, but the proportionality constant depends on the pulse shape. If the pulse shape is such that  $x_{\max} = 1$ , then  $\tau_p = 1/B_p$ .

With a pulse repetition frequency of  $R_p$ , the “duty cycle” might be defined as  $\rho = \tau_p R_p$ . The average power (transmitted or received, depending on the reference for  $P_{pk}$ ) is

$$\bar{P}_{uwb} = R_p E_p = R_p \tau_p P_{pk}. \text{ Hence, } \bar{P}_{uwb} = \rho P_{pk}.$$

Note that since the UWB receiver noise (assuming the receiver frequency response is the same as that of the pulse) is  $N_{uwb} = B_p N_0$ . If the signal-to-noise ratio is defined as  $S/N = P_{pk}/N_{uwb}$ , then

$$\frac{S}{N} = \frac{E_p}{N_0} \cdot \frac{1}{B_p \tau_p} = \frac{E_p}{N_0} \cdot x_{\max}^2$$

Thus, the relationship between  $S/N$  and  $E_p/N_0$  depends on the shape of the pulse. Note that for the UWB system,  $E_b/N_0 = (E_p/N_0) \cdot (R_p/R_b)$ . Also note that if  $x_{\max} = 1$ , then  $S/N = E_p/N_0$ , and that if  $R_b = R_p$  (one bit per pulse), then  $E_b/N_0 = E_p/N_0$ .

### **Interference to the Narrowband System and Bandwidth Correction Factors**

There are three cases that must be considered:

**Case 1:**  $R_p \leq B_{nb}$ .

In this case, the IF resolves individual pulses in time and the energy per pulse at the IF output is  $E_p \cdot B_{nb}/B_p$ , so the average interference power is

$$I = E_p R_p \frac{B_{nb}}{B_p} = P_{pk} \tau_p R_p \frac{B_{nb}}{B_p} = P_{pk} \rho \frac{B_{nb}}{B_p}$$

where  $\rho = \tau_p R_p$  is the duty cycle and the ratio  $B_{nb}/B_p$  is the bandwidth correction factor.

**Case 2:**  $R_p > B_{nb}$  and constant PRF.

In this case, the UWB spectrum consists of tones at frequencies that are harmonics of the PRF. This power in the tone at frequency  $kR_p$  is  $P_k = R_p^2 |P(kR_p)|^2$ . A tone at the frequency at which the pulse spectrum is maximum is  $P_{\max} = R_p^2 |P(f)|_{\max}^2$ , so the worst-case interference as seen by an NB receiver (with such a tone in its passband) is  $I = 2|P(f)|_{\max}^2 R_p^2$ . Since  $E_p = 2|P(f)|_{\max}^2 B_p$ , this becomes

$$I = \frac{E_p}{B_p} R_p^2 = \frac{\tau_p P_{pk}}{B_p} R_p^2 = P_{pk} \rho \frac{R_p}{B_p}$$

which is similar to the expression in case 1, except now the bandwidth correction factor is  $R_p/B_p$ .

**Case 3:**  $R_p > B_{nb}$  and the pulse positions are randomly dithered ( $R_p$  is the average rate in this case). If the pulse dithering is random, the power spectral density (PSD) of the UWB signal is  $S(f) = R_p |P(f)|^2$  and the average interference power is  $I = 2B_{nb} S(f_0) = 2R_p B_{nb} |P(f_0)|^2$ , assuming  $P(f)$  is essentially constant over the passband of the narrowband system. Since  $f_0$  is assumed to occur at the maximum of  $P(f)$ , then  $2|P(f_0)|^2 = E_p/B_p$  and the average interference within the narrowband receiver passband is:

$$I = R_p E_p \frac{B_{nb}}{B_p} = R_p \tau_p P_{pk} \frac{B_{nb}}{B_p} = P_{pk} \rho \frac{B_{nb}}{B_p}$$

as in case 1, and the bandwidth correction factor is again  $B_{nb}/B_p$ .

If there is some structure to the pulse dithering (for example, if the pulse positions are restricted to a finite number of time offsets relative to the nominal start of the UWB frame), then the UWB PSD generally will consist of a spectrally-continuous (noise-like) component and a discrete component (tones). The exact UWB interference at the NB receiver IF output would require detailed PSD analysis, as provided in [1].

## References

- [1] "Physical-Layer Modeling of UWB Interference Effects," J. E. Padgett, J. C. Koshy, and A. A. Triolo, January 10, 2003, sponsored by DARPA under contract MDA972-02-C-0056.